Jožef Stefan International Postgraduate School

Seminar

Whispering Gallery Modes

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Abstract

Because of total internal reflection light can be trapped inside a dielectric sphere that has the index of refraction larger than the outside medium forming typical resonances called Whispering Gallery Modes (WGMs). These resonances are especially interesting because they have very small mode volume and very high quality factors. WGMs can be excited either by fluorescence or by coupling the resonator to an optical fibre or prism through evanescent field extending outside the sphere. The evanescent field is a basis for all their applications. The WGMs interact with the environment outside through the evanescent field. The main applications of WGMs are biosensing, optical filters, waveguides and microlasers.

Contents

1 Optical Microcavities

Optical microcavities confine light in a very small volume. They are connected to a wide range of fields such as photonics, quantum electrodynamics (QED), atom optics and telecommunication. Possible applications are for example microlasers, narrow filters, optical switching, ultrafine sensing, displacement measurements, high resolution spectroscopy, Raman sources, enhancement and suppression of spontaneous emission and studies of nonlinear optical effects.

Resonant frequencies are size dependent. Because of small volume of the microresonator the mode volume is much smaller than in large resonators. This means that the resonant frequencies are far apart from each other thus reducing the number of exited modes. Ideal resonator would have infinitely sharp resonances and would confine light indefinitely. In real resonators this is not possible because of the losses: absorbtion and radiation loss. The *resonant Q factor* gives how much time the light is confined in the resonator in terms of optical period. From the quality factor Q factor we can calculate the spectral width of the resonance.

2 Spherical Resonators

One type of microresonators are spherical resonators made of transparent dielectric spheres[1]. If the refraction index of the sphere is grater than the index of the outside medium the light can be trapped inside the sphere as a consequence of total internal reflection (Figure 1a). If the circulating light returns to the same point in phase we get resonant standing waves (Figure 1b). This resonant modes are called *Whispering Gallery Modes* (WGMs). The name comes from Whispering Gallery - the dome of St. Catherine's cathedral in London where L. Rayleigh observed and analyzed sound bouncing of the dome walls.

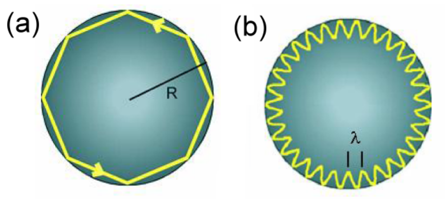

Figure 1: a) Geometrical optics and b) wave optics representation of a whispering gallery mode. [1]

WGMs are especially interesting because of their high Q values. In silica microspheres the Q factor can be as high as 8×10^9 [2]. This is possible because of very low optical loss of bulk silica and nearly perfect surface. As a starting material a commercial optical fibre is usually used. The tip of the fibre is melted with a high power laser or an oxygen-hydrogen torch. Surface tension creates nearly perfect sphere with very smooth surface. The advantage of using optical fibre is that the fibres are made from very clean materials that have very low optical losses. But the loss limit is usually determined by the surface. Just exposure to the air can significantly lower the Q value because of the formation of -OH groups on the surface. Instead of a sphere toroidal resonators are

[1] They are usually made from glass, usually silica, or polymer material like melamine formaldehyde or polystyrene.

also used. They are made from silica on a silicon wafer using lithographic methods [3]. Microtoroid resonators with Q factors as high as 10^8 were made [4].

2.1 Calculation of WGMs

We can calculate the frequencies and electric/magnetic filed distribution of the WGMs by solving Maxwell equations in spherical coordinates. From Maxwell equations in non-conducting materials with no free charge

$$\begin{aligned} \nabla \cdot \vec{E} &= 0 \quad (1) \\ \nabla \cdot \vec{B} &= 0 \quad (2) \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \quad (3) \\ \nabla \times \vec{B} &= \varepsilon\varepsilon_0\mu\mu_0 \frac{\partial \vec{E}}{\partial t} \quad (4) \end{aligned}$$

we obtain wave equation for electric field

$$\nabla^2 \vec{E} = \varepsilon\varepsilon_0\mu\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}. \quad (5)$$

Harmonic time dependence gives us the Helmholtz wave equation for electric field

$$\nabla^2 \vec{E} + k^2 \vec{E} = 0. \quad (6)$$

where $k = \frac{\omega}{c}$ is wave number. By solving the Helmholtz equation in spherical coordinates, we obtain vector spherical wave eigenfunctions. In the scalar wave equation approximation there are two classes of solutions: transversal magnetic (TM) and transversal electric (TE) case. We can separate the variables

$$E = R(r)\Phi(\phi)\Theta(\theta). \quad (7)$$

The separation of variables leads to the introduction of the radial mode number (n), the polar mode number (l) and the azimuthal mode number (m). In this respect there is quite nice analogy with the hydrogen atom, because these mode numbers correspond to quantum numbers in atoms. Therefore WGMs in a sphere are often called 'photonic atom'. The mode number n indicates the number of intensity peaks in the radial direction (Figure 2), l indicates the number of waves around the circumference of the sphere and m gives the azimuthal distribution of intensity peaks. Like in atoms the allowed azimuthal mode number m is in range of $-l < m < l$ for every l. Therefore we have $2l + 1$ degeneracy of azimuthal modes. Finally we can calculate the resonant frequency fir each set of n, l and m. The frequencies are dependent only od the radius and the refraction index of the sphere. For the modes that are confined to the equatorial plane ($l = m$) and have only one maximum in the radial direction ($n = 1$) a much more simple approximation can be used. In this case we can say that the optical path is equal to the sphere circumference $2\pi r$. On the other hand the optical path is also equal to the integer number of wavelengths $l\lambda$. So we get

$$\lambda l = 2\pi r N \quad (8)$$

where λ is the resonant wavelength, N is the refractive index of the sphere and r is the radius of the spherical resonator.

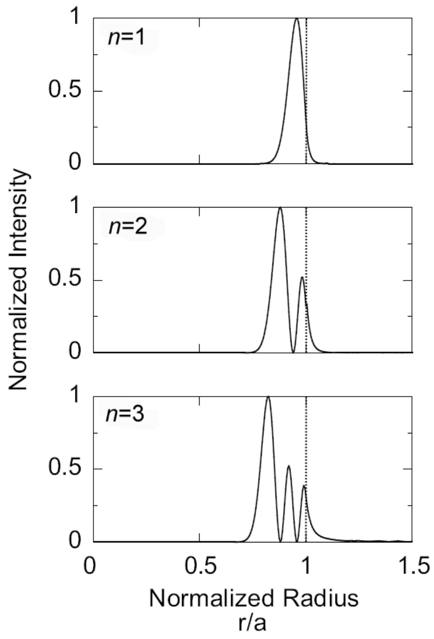

Figure 2: Radial light intensity for TE WGMs with $l = m = 60$ and $n = 1, 2$ and 3. Note that the light intensity is nonzero also outside the sphere. [5]

An important characteristics of the WGMs is that the light is not entirely confined in the sphere. Outside the sphere the light intensity exponentially decreases with distance from the surface forming the *evanescent field*. This phenomena is crucial for many applications of WGMs. Evanescent field is typically extending around one wavelength from the surface of the sphere and is non-radiative.

3 Excitation of WGMs

3.1 Evanescent field coupling

We can excite WGMs by coupling the evanescent field of the sphere with an external evanescent field. The external evanescent field can be made in two main ways. The first method is by total internal reflection of light in a prism. Because of total internal reflection we get evanescent field outside the prism. This phenomena is the same as in dielectric spheres. The second source of evanescent field can be a thin optical fibre. In both cases we need to mechanically bring the microspehere in close contact with the prism or fibre. Typical distance between the two must be under 1μm for optical wavelengths. Tunable laser is used as a light source. By selecting the wavelength that corresponds the to one WGM is it possible to selectively excite single WGMs. Usually we measure the transmission of an optical fibre coupled to a microresonator as a function of wavelength. At the wavelengths that correspond to WGMs the light energy is flowing into the sphere where there is some dissipation. At those wavelengths we get lower transmission (Figure 3).

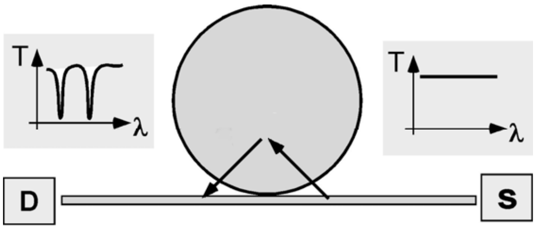

Figure 3: Spherical microresonator coupled to an optical fibre with the source on the left and the detector on the right. At resonant frequencies we get lower transmission. [6]

3.2 Fluorescence

A more simple way of exciting WGMs is fluorescence. Microspheres made from polymer material mixed with a fluorescent substance are used. As a fluorescent substance a dye or quantum dots are employed. Shorter wavelength light is used to excite the fluorescence[2]. From bulk fluorescent material or many fluorescent microspheres together we get a broad emission spectrum (Figure 5a). The spheres do not have exactly the same radii and the resonant wavelengths corresponding to WGMs are different for each sphere. So if we observe light coming from many spheres the resonant peaks are in different positions and the spectrum flattens out. The other reason that we can not see the peaks is coupling of spheres that are in contact with each other. In this case the coupled modes appear. Because of more modes, their wavelengths are closer together and we are no longer able to distinguish them from the background. In the case of a single isolated sphere peaks appear in the emission spectrum (Figure 5b). These spectral lines correspond to WGMs with different l, m and n mode numbers. The fluorescent substance in the microresonator is not more emitting light in a broad spectrum, but with preference to the wavelengths of WGMs. This phenomenon is due to enhancement and suppression of spontaneous emission.

The fluorescent substance can also be on the surface outside the microsphere (Figure 4). In this case the WGMs are excited through the coupling between the fluorescent substance and the WGMs trough evanescent field. Having the fluorescent substance near the surface is a more efficient way of exiting WGMs, because the light captured inside the sphere is circumnavigating just below the surface.

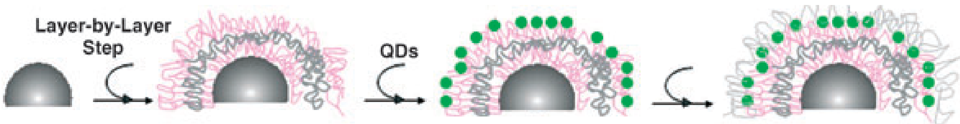

Figure 4: One convenient way of having the fluorescent substance near the surface is layer-by-layer depositions of polyelectrolytes and quantum dots. [7]

[2]For example UV lamp or laser is used.

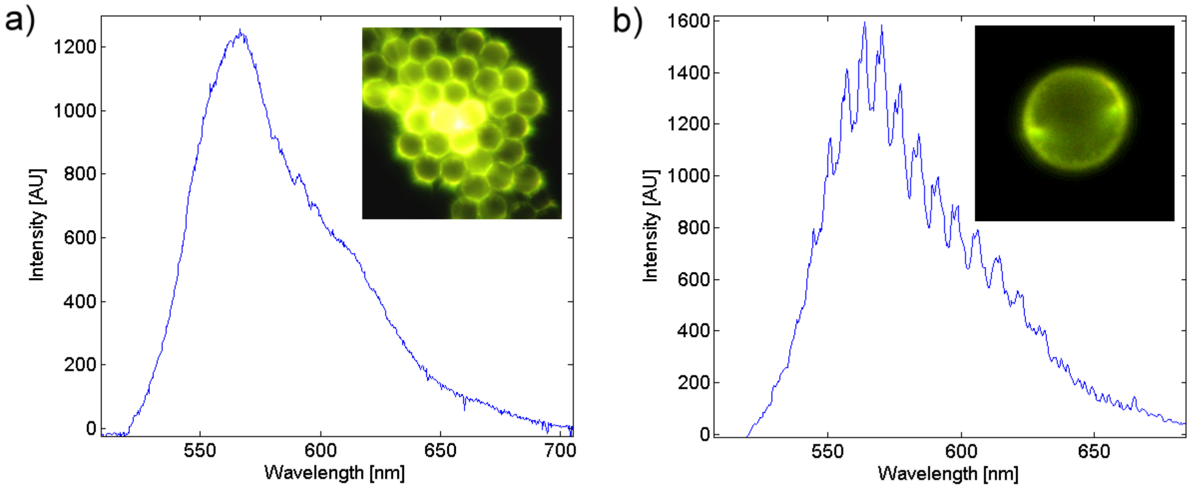

Figure 5: Fluorescence emission spectra from 10μm fluorescent polystyrene microspheres excited by 514nm Argon-ion laser. a) Spectrum from many spheres shows no distinct peaks. b) In the spectrum of a single sphere distinct resonant peaks corresponding to different modes can be seen.

4 Applications of WGMs

4.1 Biosensing

The WGMs can interact with the environment outside the microsphere through evanescent field. A particle on the surface or near the surface of a microsphere changes the optical path of the light and/or the cavity loss. This causes a shift of the resonant frequency. Because the light is circulating inside the microsphere it is sampling the particle many times. The sensitivity of such a sensor increases with increasing Q factor. Because spherical microresonators have really high Q factors they are among the most sensitive optical systems. In the case of a microresonator with $Q = 10^8$ and diameter $100\mu m$ the light travels couple of tens of meters around the sphere and the particle on the surface is sampled more than 100,000 times. In this way it is possible to detect single molecules attaching to the surface [3]. We can further functionalize the surface of the microresonator in order to bind just specific molecules to the surface. In this way selective single molecules detection is possible. Such detection has also been demonstrated in real life environment like blood serum [3].

For biosensing both evanescent field coupling and fluorescence excitation if WGMs can be used. With the fluorescence the idea is the same as for evanescent coupling. The florescence spectrum is recorded and the shifts of resonant frequencies are measured. Fluorescence is simpler, but in this way we can not achieve single molecule detection.

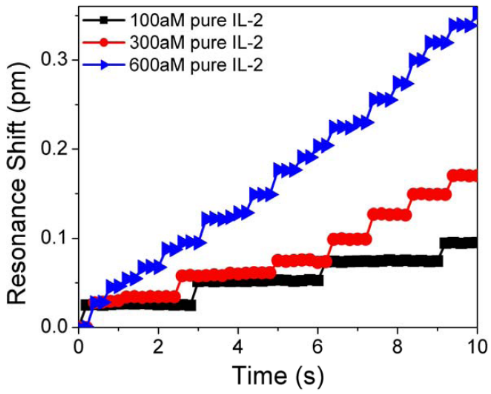

Figure 6: The graph shows the resonant wavelength shift versus time for three different target molecules concentrations. When a molecule binds to the surface the resonant wavelength changes creating steps. The binding rate is faster for higher concentrations. [3]

4.2 Filters in optical communications

The need for ever greater bandwidth in communications is continuously growing. Today optical communication is widely used for long range transmission. To increase the bandwidth of a single optical fibre more wavelengths have to be used at the same time. Therefore the interest in optical filtering and switching is gaining interest in recent years. A optical microresonator enables coupling between two optical fibres, forming a so called add/drop filter. Single frequency light matching the frequency of microresonator traveling along one fibre can be transferred with low loss to the other fibre. In the same way as a frequency is dropped from the fibre another frequency can be added (Figure 7). The advantages of a microresonator in comparison with other filters is its small size and potential for high density integration on a chip. By electrically changing the refractive index it is also possible to dynamically select the filtered frequency.

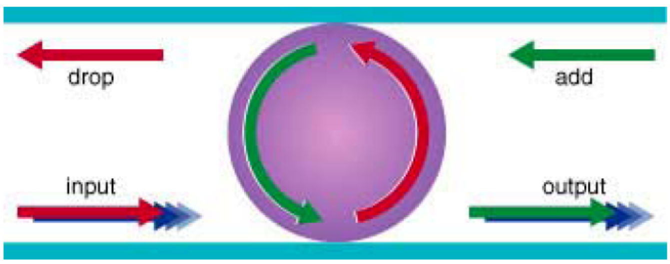

Figure 7: Microsphere coupled to two optical fibers can be used as an add-drop filter. An optical signal sent toward the sphere along top fiber (green arrow) is added to the many signals traveling along the other fiber (blue arrows). This device can also be used to extract (or drop) a signal (red arrow) that was originally traveling through a fiber along with many others. [8]

4.3 Ultralow-threshold microlasers

WGM resonators can be also used as laser sources. Lasing has been demonstrated in different microresonators such as liquid droplets, microspheres, microdisks and microcapilares. Because of small cavity volumes and high Q values submicrowatt optical pump lasing thresholds can be achieved [9]. Usually a rare earth doped silica microsphere coupled to an optical fibre is used. The frequency of the pump laser is adjusted so it matches one WGM. Above the threshold one or more peaks that correspond to WGMs appear. If we couple different microspheres to a single optical fibre a multi-wavelength laser can be constructed (Figure 8). Microresonator lasers could extend the range of available laser wavelengths especially for compact systems like semiconductor lasers.

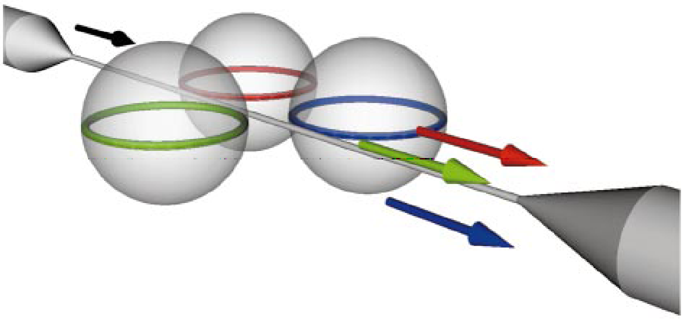

Figure 8: Three spherical microlasers coupled to a single optical fibre. [9]

4.4 Coupled resonator optical waveguides

Two microspheres near each other are coupled through evanescent field in the same way as one sphere can be coupled to prism or optical fibre. If we induce WGMs in one sphere, some light is transferred to the other sphere. By combining more spheres, we can create chain that act as waveguide (Figure 9). In contrast to other kinds of coupled resonator waveguides they possess broad bandpass transmission for signals. Such chains could be used as delay lines, optical buffers, nonlinear elements, sensors, filters and optical switches. For example, by changing the optical properties of the medium between two spheres one can change the coupling of those two spheres. Coupled resonator optical waveguides have big potential in future photonic integrated circuits.

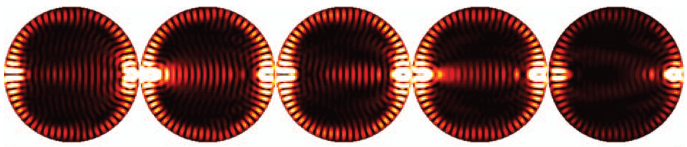

Figure 9: Chain of resonantly coupled spherical resonators. [10]

5 Conclusion

In recent years the interest in whispering gallery mode resonators is increasing rapidly. The reason are their unusually high quality factors and high energy densities making them one of the most sensitive optical systems. Together with their small size they are perfect candidates for miniaturization and integration of sensors and optical systems. Despite these advantages there is still a long way towards real life applications. In laboratories a wide range of applications were demonstrated such as microlasers, narrow filters, optical switching, ultrafine sensing, displacement measurements, high resolution spectroscopy and studies of nonlinear optical effects.

Impressive results have been achieved in so far. With new materials and microfabrication techniques there is still a lot of space for improvement. In the near future a wide range of commercial applications using WGMs is expected.

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